

The mesons and baryons production in nucleus-nucleus collisions at SPS and RHIC energies and quark-gluon plasma

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We use quasiparticle description of deconfined matter in nuclear collisions at finite temperature and chemical potential. We assume that evolution of expanding system is isentropic. Using theoretical formulas and experimental meaning for the average multiplicity of charged and neutral particles and also of net nucleons ($N - \bar{N}$) per unit central rapidity, we calculate the initial temperature T_0 , and the volume V_0 for the collisions of heavy nuclei (Pb+Pb and Au+Au) at SPS and RHIC energies. At calculations we use the conservation of entropy and of number net nucleons in initial plasma stage and on the stage of phase transition at temperature T_c , where we take into account the phase of constituent quarks m_q and m_s . We use at calculation the effective quasiparticle model with decrease of thermal gluon mass $m_g(T)$ at $T \rightarrow T_c$ from above. This model agrees with lattice data. The particle ratios for SPS and RHIC are defined by early chemical freeze-out at temperature close to T_c (at values of chemical potential $\mu_B = 247$ MeV and 50 MeV for SPS and RHIC (at $\sqrt{s} = 130$ GeV) correspondingly). The spectra of baryons and mesons are calculated at temperature of thermal freeze-out $T_f = 120$ MeV. The quantitative characteristic of these spectra (for example, the normalization for baryons and mesons) are determined in the present model by initial state - quark-gluon plasma. At calculation of spectra we assume that averaged transverse flow velocity \bar{v}_\perp have both direct particles and paternal resonance. We show that values of \bar{v}_\perp increase for energy RHIC in comparison with SPS. We had investigated also the analogous problem of nuclear collisions in quasiparticle model with phenomenological parametrization of running coupling $G(T)$, which increase at $T \rightarrow T_c$ from above (in difference from effective quasiparticle model). We have here agreement with new lattice data. We do not found in this model quantitative difference for spectra in comparison with effective quasiparticle model. However it can be shown, that in this model we have dramatic situation for energy loss of gluon jets in plasma.

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I. INTRODUCTION

There is considerable interest to heavy ion reaction at sufficiently high energies, because they provide the possibility for conditions of the transitions to deconfined state of hadronic matter-quark-gluon plasma (QGP). At Pb+Pb and Au+Au collisions there is considerable number of net nucleons ($N - \bar{N}$) in central interval of rapidity, which it is necessary take into account on plasma and hadron stage. Therefore we have the baryon chemical potential $\mu_B \neq 0$.

In the works a number of authors (P.Braun-Münzinger et.al. [1]) was shown, that statistical thermal model gives good description of particle ratios in central collisions of heavy nuclei at high energies (for SPS and RHIC). The temperature T for all this is close to critical temperature $T_c \approx 170$ MeV obtained from lattice Monte-Carlo simulations of QCD, and μ_B is close to 50 MeV for RHIC(at $\sqrt{s}=130$ GeV) This correspond to chemical freeze-out, when inelastic collisions cease to be important and the particle composition is fixed. The trajectory of chemical freeze-out $T(\mu_B)$ was obtained also in the work [2] on the basic of relation for hadronic gas $\langle E_h \rangle / \langle N_h \rangle = \text{const} \approx 1$ GeV. For SPS was found

the value $\mu_B \approx 250 - 260$ MeV. One can to suppose that trajectory $T(\mu_B)$ for various energies of nuclei is close to boundary of deconfined quark-gluon plasma phase. For all this the potential μ_s correspond to strangeness neutrality (i.e. overall strangeness = 0). This correspond in fact to hadronic part of mixed phase, where there are strange mesons and baryons (i.e. $\mu_s \neq 0$).

In the present work we try to define the physical characteristic of initial plasma phase T_0 and V_0 . For this aim we use hypothesis of conservation of entropy and of number of net nucleons in initial stage (where $\mu_s = 0$) and on stage of phase transition at $T = T_c$, where we take into account also of constituent quarks phase.

The plasma at finite temperature was considered perturbatively up to order $O(g^5)$ [3]. However in the experimentally accessible region (close to temperature T_c of phase transition) the strong coupling constant is sufficiently large: $g \sim 2$. The perturbative expansion in powers of g probably gives bad convergence. Recently in series of papers [4, 5, 6] for description of quark-gluon plasma have been used quasiparticle model, which allow apparently to sum up partially the perturbative expansion. At high temperature the plasma consist of quasifree quarks and gluons and probably is described well by perturbative theory. The lattice model show rapid increase of entropy density and other values near by T_c in SU(3) gluon plasma. In quasiparticle model the interacting plasma of quarks and gluons is described as system of

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massive quasiparticles.

For thermal momenta $k \sim T$ the quark-particle excitation and transversal gluons will propagate with dispersion relations

$$\begin{aligned}\omega_i^2(k) &\simeq m_i^2(T) + k^2, \\ m_i^2(T) &= m_{0i}^2 + \Pi_i,\end{aligned}\quad (1)$$

where Π_i are given by the asymptotic values of the hard thermal self energies [6, 7]

$$\begin{aligned}\Pi_q &= 2\omega_q(m_0 + \omega_q), \quad \omega_q^2 = \frac{N_c^2 - 1}{16N_c}[T^2 + \frac{\mu_q^2}{\pi^2}]G^2, \\ \Pi_g &= \frac{1}{6}[(N_c + \frac{N_f}{2})T^2 + \frac{6}{2\pi^2}\mu_q^2]G^2.\end{aligned}\quad (2)$$

The contributions Π_i are generated dynamically by the interaction within the medium, and $m_i(T)$ acts as effective mass. The quantity $G^2(T)$ is to be considered as effective coupling in quasiparticles model. The entropy and energy density for example for gluon plasma takes the form:

$$s_g(T) = \frac{g_g}{2\pi^2 T} \int_0^\infty k^2 f(E_k) \frac{4k^2 + 3m_g^2(T)}{3E_k} dk \quad (3)$$

$$\epsilon_g(T) = \frac{g_g}{2\pi^2} \int_0^\infty k^2 f(E_k) (E_k) dk + B(T), \quad (4)$$

where $f(E_k) = [\exp(\frac{E_k}{T}) - 1]^{-1}$, $E_k = \sqrt{k^2 + m_g^2(T)}$

The value $B(T)$, introduced in expression for energy (and also for pressure) is necessary in order to maintain thermodynamic consistency, that is must be fulfilled the relations $\epsilon + p = sT$ and $s = \frac{\partial \epsilon}{\partial T}$. The function $B(T)$ compensates the additional T derivatives from $m(T)$ in pressure and it represent in integral form in the paper [5]. In our paper we do not use the function $B(T)$.

However the phenomenological parameterization of coupling constant G^2 in accordance with perturbative QCD in quasiparticle model [4, 6] is very likely unjust close to temperature of phase transition T_c , where it should be expected of nonperturbative dynamics. In this model we have the increase of thermal gluon mass in vicinity of phase transition.

In Sec. II we give the brief review of phenomenological model of confinement [10], which agrees with $SU(3)$ lattice data. This model is applied also to system with dynamical quarks.

In Sec. III we calculate for isentropic evolution of quasiparticles system with chemical potential $\mu_B \neq 0$ the various physical characteristic: the initial temperature T_0 , the volume V_0 , the initial entropy S_0 etc. for nuclear collisions at SPS and RHIC energies. We use the values T_c and μ_B close to above-mentioned: $T_c = 170$ MeV and $\mu_B = 50$ MeV and 247 MeV for RHIC and SPS. At calculation we use also additional information - experimental meaning of average number of net nucleons in central

region of rapidity for SPS and RHIC. In result we have obtained the equations (22), (23) for definition of T_0 . Simultaneously must be fulfilled the relations (24), which follow from conservation of entropy and of number net nucleons.

Besides of study of characteristic of initial phase we describe in Sec. III another new aspect - we show, that from conservation of entropy and of number of net nucleons follow, that massive constituent quarks (m_q and m_s) appears with decrease of number of degrees of freedom (i.e. with effective number) in the presence of octet of pseudogoldstone states. We show also, that with the same effective number of degrees of freedom appears hadrons and resonances in hadron part of mixed phase (for SPS and RHIC).

Certainly it should be noted, that in nucleus-nucleus collisions at high energies there is possibility of two stages of equilibrium. The gluons reach of thermal equilibrium to a considerable extent faster, than quarks. According to estimations, equilibrium time of gluons is $\tau_g \sim 0.3$ fm/c and for quarks one is $\tau_q \sim 2$ fm/c, i.e. it is possible the production of hot glue at first stage [11]. The corresponding estimations of initial conditions for hot glue in effective quasiparticle model there are in the work [12]

In Sec. III we investigate the initial condition (T_0, V_0) and evolution of second plasma stage, where is the equilibrium also for quarks. In this paper we do not consider the particle ratios. We have considered the ratios for various baryons and mesons with accounting of resonance decays in the work [33]. The particle ratios in the main agrees well with experimental data for SPS and RHIC ($\sqrt{s} = 130$ MeV).

In Sec. IV we calculate in effective quasiparticle model the some baryons and mesons (pions and kaons) spectra with accounting of resonance decays and transverse flow. We use for heavy nuclei the thermal freeze-out temperature $T_f = 120$ MeV. The normalization of baryons and mesons spectra here is no free parameter, but is defined by initial condition in plasma - by initial entropy S_0 and by condition of entropy conservation $S_f = S_0$. We show that values of averaged transverse flow velocity \bar{v}_\perp increase for RHIC in comparison with SPS.

The main object of present work is no study the whole of spectra for SPS and RHIC, but study of dependence spectra from initial state and from stage of phase transition plasma into hadrons.

Therefore in Sec. V we consider the analogous isentropic problem of nuclear collisions by the use of ordinary perturbative theory up to order $O(\alpha_s(T))$ with QCD parameter $\lambda \sim 0.2$ GeV. We calculate here also the values T_0, V_0, S_0 . We do not find noticeable difference for spectra of particles in comparison with effective quasiparticle model. However this model disagrees with $SU(3)$ lattice data in region of phase transition (i.e. near of temperature T_c). In Sec. V we consider also the quasiparticle model with phenomenological parametrization of running coupling $G(T)$, which gives good fit of new lattice data. We calculate here also the initial parameteres T_0, V_0, S_0 .

The coupling strength G_T here increase at $T \rightarrow T_c$ from above (unlike effective quasiparticle model). However baryons and mesons spectra in this model practically do not differ from spectra in effective quasiparticle model. Thus the spectra weakly depend on region of phase transition.

However it can be shown [12], that in ordinary perturbative model (taking into account the hot glue) we have too great energy loss ΔE_g of gluon jet at RHIC energy: $\sim 80 - 90\%$ (unlike effective quasiparticle model). Thus the jet quenching essentially depend on description of phase transition stage. We show also, that in second above-mentioned model the situation with energy loss ΔE_g is more dramatic.

In Sec. VI - conclusion.

II. EFFECTIVE QUASIPARTICLE MODEL

In the papers [4, 6] for gluon plasma was used the phenomenological parameterization of coupling constant $G^2(T)$ in accordance with perturbative QCD with the two fit parameters T_s and λ . The resulting equation of state (EOS) is in good agreement with lattice data over a wide temperature range between T_c and $5T_c$. However in spite of this the thermal gluon mass have strong increase in the vicinity of the phase transition at decrease of T . Furthermore, close to a phase transition the coupling constant $G_s(T_c) \sim 2$, that is correctness of perturbative of calculations is questionable.

Further it is expected that correlation length $\xi(T)$, which is proportional m_D^{-1} , grows when $T \rightarrow T_c$ from above (m_D is gluonic Debye mass). The lattice calculations shows that m_D drops by factor of 10 when T decrease from $2T_c$ to T_c [8]. Also for three colors the value $\xi(T_c)$ remains large, but finite. In hard thermal loop (HTL) perturbation theory the thermal mass $m_g(T)$ and gluonic Debye mass m_D are connected by relation [9]

$$m_D = \sqrt{2} m_g. \quad (5)$$

However in nonperturbative theory this relation can be broken down [9].

Recently in the paper [10] was considered interesting phenomenological model of confinement. In region below T_c in a pure $SU(3)$ gauge theory are color singlet of heavy glueballs. Approaching T_c the gluons are liberated, followed by sudden increase of energy and entropy density. When approaching the phase transition from above, the decrease of thermodynamic values is not caused by increase of masses(unlike [4, 6]), but caused by decrease of number active degrees of freedom. When T comes closer to T_c , more and more of gluons are absorbed by glueballs. It is assumed that thermal gluons mass $m_g(T)$ follows roughly the behavior of the Debye mass, i.e. it decreases. However the entropy density $s(T)$ will overshoot the lattice entropy because light masses near T_c . This difference may be accounted in quasiparticle model

by modifying of number effective degrees of freedom in (3), (4)

$$g_g \rightarrow C(T)g_g. \quad (6)$$

The explicit value $C(T)$ may be estimated as the ratio of the lattice entropy and entropy (3) with a dropping mass $m_g(T)$. At $T \gg T_c$ we expect $C(T) \simeq 1$ and near $T \sim T_c$ we have $C(T) < 1$. At $T < T_c$ it can estimate $C(T_c) \sim 0.2$ from lattice data. The value $C(T)$ is a smooth, monotonously increasing function with T .

It is assumed also that thermal mass (for example gluons mass) have form $m_g(T) = \tilde{G}(T)T$. The lattice results for Debye mass $m_D(T)$ can be parameterized well by formula

$$m_D(T) \simeq G_1 T \left(1 + \delta - \frac{T_c}{T}\right)^\beta \equiv \left(\frac{N_c}{6}\right)^{1/2} G_0 T \left(1 + \delta - \frac{T_c}{T}\right)^\beta \quad (7)$$

where $G_1 \simeq 1.3$, $\beta \simeq 0.1$ and there is small gap at $T = T_c$ with $\delta \simeq 10^{-6}$. The value G_1 is determined here by the asymptotic value of thermal gluon mass, which coincide with lattice mass for instance at $T = 3T_c$. Below of this value the explicit HTL resummation is expected to fail [13]. One can to use the asymptotic value $G(T)$ with renormalization group inspired parametrization [6]:

$$G^2(T, \mu = 0) = \frac{24\pi^2}{(11N_c - 2N_f) \ln(\lambda(T + T_s)/T_c)} \quad (8)$$

The parameteres T_s and λ are used for fitting new lattice results with $N_f=2$ and $N_f=4$ at $T = 3T_c$ (and also for $N_f=3$ [6]). The thermal gluon mass at chemical potential $\mu_q=0$ is analogous with formula (7) where $G_1 \rightarrow \sqrt{N_c/6 + N_f/12G_0}$. Effective coupling G_0 have the form $G_0 = \frac{g_0}{\sqrt{11N_c - 2N_f}}$. The gluon and quark mass coincide with lattice data at $g_0 \simeq 9.4$ for $N_f=2$ and at $g_0 \simeq 9.8$ for $N_f=3$. That gives $G_0 \simeq 1.886$ for $N_f=3$. The like value $G_0=1.9$ we have in formula (7). The such value $G_0 \simeq 1.9$ we use in calculations. It should be noted, that lattice results at finite chemical potential μ_q point to greatly weak dependence of asymptotic thermal masses from μ_q [14].

Since (in accord with assumption) the values $m_g(T)$ and $m_D(T)$ have similar trends near $T \sim T_c$, the mass $m_g(T)$ is parameterized by analogy with formula (7). In order to account for certainties, were investigated [10] some a range of values G_0 , β and δ .

A decreasing effective coupling strength $G(T)$ at $T \rightarrow T_c$ from above can be understood, since at decreasing of T more and more gluons become confined and form heavy glueballs. The effective glueball exchange interaction between gluons are reduced. The total interaction can be interpreted as a superposition nonperturbative multigluon and weak glueball exchange. The good fit [10] for $C(T)$ (6) have form similar with (7)

$$C(T, T_c) = C_0 \left(1 + \delta_c - \frac{T_c}{T}\right)^{\beta_c}, \quad (9)$$

where $C_0 \simeq 1.25$, $\delta_c \simeq 0.0026$, $\beta_c \simeq 0.31$.

The relations (7), (9) give good description of $SU(3)$ lattice data for ϵ/T^4 , $3s/4T^3$, $3p/T^4$ for T close to T_c .

Further it is possible to extrapolate effective quasiparticle model to system with dynamical quarks. Unfortunately no lattice data on thermal masses with dynamical quarks are available. We assume that N_c and N_f dependence of m_g and m_q are given by formulas (2), where effective coupling $G(T)$ have the form: $G(T) = G_0(1 + \delta - T_c/T)^\beta$. For example, the thermal masses of u and d quarks are:

$$\begin{aligned} m_q^2 &= m_{q0}^2 + 2m_{q0}G(T)\sqrt{\frac{N_c^2 - 1}{16N_c}(T^2 + \frac{\mu_q^2}{\pi^2})} + (10) \\ &+ 2\frac{N_c^2 - 1}{16N_c}(T^2 + \frac{\mu_q^2}{\pi^2})G^2(T). \end{aligned}$$

For s quarks we replace $m_{q0} \rightarrow m_{s0}$, $\mu_q \rightarrow 0$. It is assumed that parameterization of function $C(T, T_c)$ in the presence of quarks and parameterization (9) for gluons are similar, with some variation of parameters. For example, $C_0 = 1.25$, $\delta_c \simeq 0.02$, $\beta_c \simeq 0.28$ for two light quarks [10]. The relations of type (9) for $C(T, T_c)$ and relations of type (10) for m_q and m_g are inserted into formulas for entropy density of quasiparticles. For $q(u, d)$ quarks we have the entropy density:

$$\begin{aligned} s_q(G_0, m_q, \mu_q, T, T_c) &= (11) \\ &= s_{q1}(G_0, m_q, \mu_q, T, T_c) + \\ &+ s_{q1}(G_0, m_q, -\mu_q, T, T_c) - \\ &- \frac{\mu_q T^2 g_q}{2\pi^2} (F_1(G_0, m_q, \mu_q, T, T_c) - \\ &- F_1(G_0, m_q, -\mu_q, T, T_c)) \end{aligned}$$

where

$$\begin{aligned} s_{q1}(G_0, m_q, \mu_q, T, T_c) &= \\ &= \frac{T^3 g_q}{2\pi^2} \int_0^\infty dx \frac{(4x^4 + 3x^2 \frac{m_q^2(T)}{T^2})C(T, T_c)}{3\sqrt{x^2 + \frac{m_q^2(T)}{T^2}} [e^{\sqrt{x^2 + \frac{m_q^2(T)}{T^2}} - \frac{\mu_q}{T}} + 1]} \end{aligned}$$

The formula (11) gives correct term already in order g^2 at perturbative expansion.

For s quarks $m_q \rightarrow m_s$, and $\mu_s = 0$. For gluons the entropy density is:

$$\begin{aligned} s_g(G_0, m_g, T, T_c) &= (12) \\ &= \frac{T^3 g_g}{2\pi^2} \int_0^\infty dx \frac{(4x^4 + 3x^2 \frac{m_g^2(T)}{T^2})C(T, T_c)}{3\sqrt{x^2 + \frac{m_g^2(T)}{T^2}} (e^{\sqrt{x^2 + \frac{m_g^2(T)}{T^2}}} - 1)}. \end{aligned}$$

For the net nucleons density we have:

$$\begin{aligned} n_1(G_0, m_q, \mu_q, T, T_c) &= (13) \\ &= \frac{T^3}{6\pi^2} g_q [F_1(G_0, m_q, \mu_q, T, T_c) - F_1(G_0, m_q, -\mu_q, T, T_c)], \end{aligned}$$

where

$$F_1(G_0, m_q, \mu_q, T, T_c) = \int_0^\infty dx \frac{x^2 C(T, T_c)}{e^{\sqrt{x^2 + \frac{m_q^2(T)}{T^2}} - \frac{\mu_q}{T}} + 1}.$$

We shall show below that in massive constituent quarks phase and hadron part of mixed phase we have also decrease of the number of effective degrees of freedom.

III. INVESTIGATION OF PHYSICAL CHARACTERISTIC OF INITIAL AND MIXED PHASES

In "standard model" of relativistic nuclear collisions after a pre-equilibrium period the system becomes thermalised in form of quark-gluon plasma with formation time τ_0 . The system then expand, cools off and at the time T_c it begins to hadronize. At temperature T_c appears the mixed phase — at first plasma part at T_c , then hadronic part of mixed phase at τ_H . At temperature near to $T_c \simeq 170$ MeV particles first undergo a chemical freeze out, and then a thermal freeze out at temperature $T_f \sim 100 - 140$ MeV. The particles ratio is defined by temperature close to T_c . For $A - A$ collisions we have the initial volume $V_0 = \pi R_A^2 \tau_0$, where $R_A \simeq 1.2 A^{1/3}$. We consider isentropic evolution. The entropy per unit rapidity is conserved during the expansion [16, 17]. The value $\frac{ds}{dy} = \pi R_A^2 s(\tau) \tau = \text{const}(\frac{dN}{dy})_{y=0}$ determine initial entropy density, where $\frac{dN}{dy}$ is the average multiplicity charged plus neutral particles per unit central rapidity. In this case part of initial energy goes into collective motion of the expanding system, that is the initial energy density must have been higher than in the free flow scenario. In the work [18] was investigated the isentropic longitudinal expansion of ideal gas of gluons and three flavors of massless quarks. In this work was found the dependence $\epsilon_0 \simeq (\frac{dN}{dy})^{4/3}$ for initial energy density in plasma. In this paper we have investigated more common case for quasiparticles with $\mu_B \neq 0$. The quasiparticles density in plasma phase is:

$$n_0 = n_q + n_s + n_g, \quad (14)$$

where n_q, n_s, n_g is density of u, d, s quarks and gluons. We have here:

$$\begin{aligned} n_q(G_0, m_q, \mu_q, T_0, T_c) &= (15) \\ &= \frac{g_q T_0^3}{2\pi^2} [F_1(G_0, m_q, \mu_q, T_0, T_c) + F_1(G_0, m_q, -\mu_q, T_0, T_c)], \end{aligned}$$

$$n_s(G_0, m_s, T_0, T_c) = \frac{g_s T_0^3}{2\pi^2} \int_0^\infty dx \frac{x^2 C(T_0, T_c)}{e^{\sqrt{x^2 + \frac{m_s^2(T_0)}{T_0^2}}} - 1}, \quad (16)$$

$$n_g(G_0, m_g, T_0, T_c) = \frac{g_g T_0^3}{2\pi^2} \int_0^\infty dx \frac{x^2 C(T_0, T_c)}{e^{\sqrt{x^2 + \frac{m_g^2(T_0)}{T_0^2}}} - 1}. \quad (17)$$

By analogy it can be written for summary entropy density:

$$s_0 = s_q + s_s + s_g, \quad (18)$$

where s_q, s_s, s_g given by (11), (12). We must express the initial temperature T_0 across the number of secondary particles per unit central rapidity. For convenience of calculations we introduce the designation: $n_0 \equiv T_0^3 n_{01}$, $s_0 \equiv T_0^3 s_{01}$, $\epsilon_0 \equiv T_0^4 \epsilon_{01}$, that is we single out the factors T_0^3, T_0^4 . We single out also the constant factors $n_{01} \equiv 12n_{02}/(2\pi^2)$, $\epsilon_{01} \equiv 12\epsilon_{02}/(2\pi^2)$. Thus we have $T_0 = (n_0/n_{01})^{1/3}$, $\epsilon_0 = (n_0/n_{01})^{4/3} 12\epsilon_{02}/(2\pi^2)$. Hence we have here $\epsilon_0 \equiv \left(\frac{N}{V_0}\right)^{4/3} f$, where $f = \frac{\epsilon_{02}(2\pi^2/12)^{1/3}}{(n_{02})^{4/3}}$.

Thus the initial temperature T_0 have been expressed across the number of secondary per unit central rapidity:

$$T_0 = \left(\frac{dN}{dyV_0}\right)^{1/3} \left(\frac{2\pi^2}{12n_{02}}\right)^{1/3}. \quad (19)$$

For $A - A$ collisions the value $\frac{dN}{dy}$ can be extrapolated by the form [19]:

$$\left(\frac{dN}{dy}\right)_{AA} = \left(\frac{dN}{dy}\right)_{pp} A^\alpha. \quad (20)$$

For pp collisions we have good approximation per unit central rapidity [20]:

$$\left(\frac{dN}{dy}\right)_{y=0} \simeq 0.8 \ln \sqrt{s}. \quad (21)$$

The value α describes the amount of rescattering in the interaction of the nuclei. From $p - A$ data it can expect $\alpha \simeq 1.1$. The value $\alpha = 1.1$ describes the result for various light ion quite well [19]. The data for SPS ($\sqrt{s} = 17.2$ GeV) is $\alpha = 1.08 \pm 0.06$ (i.e. close to $(p - A)$ data). We have $\frac{dN}{dy} \simeq 803$ (for $\alpha = 1.1$). The data of Phenix collaboration show increase of $\frac{dN_{ch}}{dy}$ from SPS (Na49) to RHIC ($\sqrt{s} = 130$ GeV) on value $\simeq 1.7$, i.e. on value $\frac{\ln(\sqrt{s}=130)}{\ln(\sqrt{s}=17.2)}$. For SPS we assume $\alpha = 1.1$. From formula (20) we have for RHIC ($A = 197$): $\alpha \simeq 1.11$.

The initial volume V_0 in formula (19) can be expressed across experimental meaning of average number of net nucleons $N - \bar{N}$ in central region of rapidity. For SPS the number of net protons in central region of rapidity is estimated as 28-29 [21]. We use the value $N - \bar{N} = 57$. The number of net protons for RHIC in central region of rapidity (at $\sqrt{s} = 130$ GeV) is estimated as 8-10 [22], i.e. the number of net nucleons is $\sim 16-20$. We use the value $N - \bar{N} \simeq 16.3$ (for some greater agreement with spectra).

We have: $N - \bar{N} = V_0 n_1(G_0, m_q, \mu_q, T, T_c)$ (from (13)). Hence the value T_0 is:

$$T_0 = \left(\frac{dN}{dy} \frac{n_1}{N - \bar{N}}\right)^{1/3} \left(\frac{2\pi^2}{12n_{02}}\right)^{1/3}. \quad (22)$$

We single out here the factor T_0^3 in formula (13) for n_1 and take into account the values $\frac{dN}{dy} \simeq 803$ and $N - \bar{N} \simeq 57$ for SPS and $\frac{dN}{dy} \simeq 1374$ and $N - \bar{N} \simeq 16.3$ for RHIC.

In result we have the equation for definition of initial temperature T_0 :

$$D_0 \equiv a_0 \left(\frac{d_2(G_0, m_q, \mu_q, T_0, T_c)}{n_{02}(G_0, m_q, \mu_q, T_0, T_c, m_s)}\right)^{1/3} = 1, \quad (23)$$

where

$$\begin{aligned} d_2(G_0, m_q, \mu_q, T_0, T_c) &\equiv \\ &\equiv (F_1(G_0, m_q, \mu_q, T_0, T_c) - F_1(G_0, m_q, -\mu_q, T_0, T_c)). \end{aligned}$$

For calculations of values n_{02} we use formulas (14-17)

For SPS we have with accounting of constant factors: $a_0 = 1.676$ at $\alpha = 1.1$ and for RHIC $a_0 = 3.045$ at $\alpha = 1.11$. We have from here the estimations of initial temperature $T_0 \simeq 175$ MeV for SPS and $T_0 \simeq 219.6$ MeV for RHIC.

We use here the meaning for temperature of phase transition $T_c = 170$ MeV, as indicated by result from lattice gauge theory [23], and close to T_c the temperature of chemical freeze-out. At SPS energies we have for value $\mu_B = 247$ MeV the good approximation for different antiparticles and particles ratios. For RHIC ($\sqrt{s} = 130$ GeV) we use also $T_c = 170$ MeV and $\mu_B = 50$ MeV. That give the ratio $\bar{p}/p \approx 0.56$ close to found before experimental value. Later was given [24] the ratio $\bar{p}/p = 0.6 \pm 0.04 \pm 0.06$. In the work [1] were given more precise values $\mu_B = 46$ MeV, $T_c = 174$ MeV and particle ratios (at $\sqrt{s} = 130$ GeV). However there are for the present noticeable statistic and systematic mistake in data. One should that not great distinction have weak influence for example on spectra baryons and mesons.

We use also the conservation of entropy and of number net nucleons in initial phase at $T = T_0, V = V_0$ and in mixed phase at $T = T_c, V = V_c$ (the entropy density is s_c): $s_0(T_0)V_0 = s_c(T_c)V_c$, $n_1(T_0)V_0 = n_1(T_c)V_c = N_{net} = N - \bar{N}$, i.e. $V_0 = \frac{N - \bar{N}}{n_1(T_0)}$, $V_c = \frac{N - \bar{N}}{n_1(T_c)}$.

The entropy density $s_c(T_c)$ correspond to sum (18), where $T_0 \rightarrow T_c$. Hence we have relations:

$$\frac{s_0(T_0)}{n_1(T_0)} = \frac{s_c(T_c)}{n_1(T_c)}. \quad (24)$$

These relations depend also from constant G_0 . The relations (23) and (24) must be fulfilled simultaneously. For SPS ($\mu_B = 247$ MeV, $T_c = 170$ MeV) these relations indeed are fulfilled. It can be seen from mentioned below values. For example for $\alpha = 1.1$ at $G_0 \simeq 1.9$, $T_0 \simeq 175$ MeV we have: $\frac{s_0(T_0)}{n_1(T_0)} = \frac{s_c(T_c)}{n_1(T_c)} = 61.4$. It should be noted, that relations (24) are fulfilled only for coincides values μ_B at initial and mixed phases. Thus, in this model the initial temperature T_0 at SPS energy exceed weakly the temperature T_c of phase transition. We will express all physical values in units $m_\pi = 139$ MeV $\simeq \frac{1}{1.42} \text{ fm}^{-1}$.

From formulas (11)-(13) - we find the values of entropy density for SPS at T_0 and T_c : $s_0(T_0) \simeq 19.43 m_\pi^3$, $s_c(T_c) \simeq 15.31 m_\pi^3$ and also net nucleons density $n_1(T_0) = 0.314 m_\pi^3$, $n_1(T_c) = 0.25 m_\pi^3$.

For the number of net nucleons $(N - \bar{N}) = n_1(T_0)V_0 \simeq 57$ we find the volume of initial plasma phase: $V_0 \simeq 181.3 m_\pi^{-3} \simeq 518 \text{ fm}^3$ (i.e. $\tau_0 \simeq 3.28 \text{ fm}$). The initial entropy is: $S_0 = s_0 V_0 \simeq 3508$. We have also the volume V_c of plasma part of mixed phase, taking into account the conservation of entropy and of net nucleons: $V_c = \frac{s_0 V_0}{s_c(T_c)} = \frac{N - \bar{N}}{n_1(T_c)} \simeq 228 m_\pi^{-3} \simeq 650 \text{ fm}^3$ (i.e. $\tau_c \simeq 4.1 \text{ fm}$).

Therefore with accounting of valent u, d quarks we have the large initial $\tau_0 > 1 \text{ fm}$. The suitable calculations by formula (19) for $\tau_0 = 1 \text{ fm}$ (for SPS) shows that for various values of G_0 the number 57 of net nucleons correspond $\mu_B \simeq 330 \text{ MeV}$, that contradict to experimental data for particles ratios. For correct values of $\mu_B \sim 200 - 250 \text{ MeV}$ the number $N - \bar{N}$ is too little ($\sim (35 - 45)$). The analogous situation there is and for RHIC.

At decrease of interaction the thermal masses of quasi-particles $m_q(T)$ and $m_s(T)$ (10) are diminished at $T \rightarrow T_c$ from above, but they do not correspond to constituent masses of quarks in mixed phase. For example, in the papers [25, 26] was shown, that by means of help of masses constituent quarks $m_u = m_d = 363 \text{ MeV}$, $m_s = 538 \text{ MeV}$ can be found the masses of baryons in octet. For $SU(3)$ group the spontaneous breaking of chiral symmetry and appearance of masses leads to appearance of octet pseudoscalar light Goldstone (or rather pseudogoldstone) states π, k, η [27]. However if in place of relation $\frac{s(T_c)}{n_1(T_c)}$ in (24) to use relations for massive quarks with accounting of pseudogoldstone states, then these relations do not fulfilled. But these relations will be satisfied, if to use decrease of the number of effective degrees of freedom of massive quarks. We calculate the entropy and net nucleon density s_c and n_c of massive quarks by formulas for ideal gas. For example for $m_u = 363$, $m_s = 538 \text{ MeV}$ at $\mu_B = 247 \text{ MeV}$, $T_c = 170 \text{ MeV}$ we have $s_c(T_c) = 14.77 m_\pi^3$, and $n_c(T_c) = 0.33 m_\pi^3$.

For entropy density s_{ps} of pseudogoldstone states $s_{ps} = s_\pi + s_k + s_\eta$ we find by same way: $s_{ps} \simeq 3.76 m_\pi^3$ (at $T_c = 170 \text{ MeV}$, $m_\pi \simeq 135 \text{ MeV}$). The factor β_1 of decrease of number degrees of freedom can be defined from relation $\frac{s_c \beta_1 + s_{ps}}{n_c \beta_1} \simeq 61.4$. That gives $\beta_1 \simeq 0.685$.

Therefore, it is possible to interpret, that there is appearance of massive constituent quarks with effective number of degrees of freedom in the presence of octet of pseudogoldstone states, while the multigluon state is absorbed into constituent quarks masses.

Let us list appropriate calculations for RHIC ($\sqrt{s} = 130 \text{ GeV}$). For example, at $\alpha = 1.11$ (20), $G_0 = 1.9$, $T_c = 170 \text{ MeV}$ we find from (23): $T_0 \simeq 219.6 \text{ MeV}$, that is from here: $s_0(T_0) = 56.44$, $s_c(T_c) = 14.8$, $n_1(T_0) = 0.145$, $n_1(T_c) = 0.048$ in units of m_π^3 .

The initial volume $V_0 \simeq \frac{16.3}{n_1(T_0)} \simeq 112.5 m_\pi^{-3} \simeq 322.5 \text{ fm}^3$, that is $\tau_0 \simeq 2.2 \text{ fm}$ (for $\pi(R_{Au})^2 \simeq 148 \text{ fm}^2$). The initial entropy $S_0 = s_0(T_0)V_0 \simeq 6350$.

Hence we have for RHIC $\frac{s_0(T_0)}{n_1(T_0)} \simeq 389$, $\frac{s_c(T_c)}{n_1(T_c)} \simeq 308$, that is the relations (24) do not fulfilled. However one can assume that at sufficiently high initial temperature $T_0 > T_c$ it is possible already the appearance (in non-perturbative state at $T = T_c$) of pseudogoldstone with entropy density s_{ps} . It gives in fact $\frac{s(T_c) + s_{ps}}{n_1(T_c)} \simeq 387$. That is we have on this stage of mixed phase for RHIC $s_0 V_0 \simeq (s(T_c) + s_{ps}) V_c$, where $V_c = \frac{N - \bar{N}}{n_1(T_c)} \simeq 339 m_\pi^{-3}$, i.e. $\tau_c \simeq 6.5 \text{ fm}$.

At high energy (RHIC) the multigluon state is also absorbed into massive constituent quarks with lesser number of degrees of freedom. Can be found the factor β_3 of decrease of effective number of degrees of freedom. We calculate here by analogy with SPS the entropy density s_{c1} and net nucleon density n_{c1} for massive quarks $m_{u,d} = 363 \text{ MeV}$, $m_s = 538 \text{ MeV}$ at $\mu_B = 50 \text{ MeV}$, $T_c = 170 \text{ MeV}$. We find in result: $s_{c1} = 14.09 m_\pi^3$ and $n_{c1} = 0.0643 m_\pi^3$. We must have ratio $\frac{s_{c1} \beta_3 + s_{ps}}{n_{c1} \beta_3} \simeq 390$. We find from here $\beta_3 \simeq 0.343$. We have effective volume V_{c1} on the stage of constituent quarks $V_c = \frac{s_0 V_0}{s_{c1} \beta_3 + s_{ps}} \simeq 739 m_\pi^{-3}$, or $V_c = \frac{N - \bar{N}}{n_{c1}(T_c) \beta_3} \simeq 739 m_\pi^{-3}$ - is the same.

Let us consider now the hadronic part of mixed phase. We have seen, that entropy density of massive constituent quarks is defined by effective number of degrees of freedom and by contribution of octet pseudogoldstone. It can be shown that with such effective number of degrees of freedom (β_1 and β_3) in hadron part of mixed phase appears nucleons and hadrons. In order to find strange chemical potential μ_s for hadron part of mixed phase, we use condition of disappearance of strangeness:

$$\sum_i n_s^i - \sum_i n_{\bar{s}}^i = 0 \quad (25)$$

that is: $\sum_i g_i (F_{1i} - \bar{F}_{1i}) - \sum_j g_j (F_{2j} - \bar{F}_{2j}) s = 0$. Here

$$F_{1i} = \int_0^\infty dx \frac{x^2}{e^{\sqrt{x^2 + \frac{m_i^2}{T^2} - \frac{\mu_s}{T}}} - 1},$$

$$\bar{F}_{1i} = F_{1i}(\mu_s \rightarrow -\mu_s),$$

$$F_{2j} = \int_0^\infty dx \frac{x^2}{e^{\sqrt{x^2 + \frac{m_j^2}{T^2} - \frac{\mu_B - s\mu_s}{T}}} + 1},$$

$$\bar{F}_{2j} = F_{2j}(\mu_B - s\mu_s \rightarrow -\mu_B + s\mu_s).$$

For strange baryons we have $s = 1$ except $\Xi(s = 2)$ and $\Omega(s = 3)$. At temperature T_c one should take into

account the considerable number of resonances. We took into account the strange mesons up to $m_k^* = 1820$ MeV and the strange baryons up to $m_B = 1940$ MeV. In result of calculations we have from (25): $\mu_s \simeq 61.44$ MeV at $\mu_B = 247$ MeV and $\mu_s \simeq 11.23$ MeV at $\mu_B = 50$ MeV (for $T_c = 170$ MeV).

At SPS we find the entropy density according to calculation by formulas of ideal gas: $\simeq 5.44m_\pi^3$ for nonstrange mesons and $\simeq 3.26m_\pi^3$ for strange ones (at T_c). For summary entropy density of strange baryons and the whole of nonstrange ones $n + p + \Delta + N_{440}^* + \dots$ we find: $\simeq 5.82m_\pi^3$, i.e. the whole entropy is: $s_H \simeq 14.52m_\pi^3$. For net nucleons density with accounting of nucleonic resonances N^* the calculation gives: $n_H \simeq 0.402m_\pi^3$. Hence the ratio is: $\frac{s_H}{n_H} \simeq 36.2$, i.e. $\frac{s_H}{n_H} < \frac{s_c}{n_c}$. Taking into account only nucleons (without of resonances), we have $\frac{s_H}{n_H} = \frac{11.5}{0.1316} \simeq 87.3 > \frac{s_c}{n_c}$. For $N + \Delta$ we have $\frac{s_H}{n_H} \simeq 46.5$, for $N + \Delta + N_{1440}^* + N_{1520}^* + N_{1535}^*$ the calculation gives $s_H = 13.19m_\pi^3$, $n_H = 0.303m_\pi^3$, i.e. $\frac{s_H}{n_H} \simeq 44$ and so on. Thus always $\frac{s_H}{n_H} \neq \frac{s_c}{n_c}$, i.e. the relations (24) do not fulfilled.

However for effective number of nucleon resonances $N + \Delta + N_{1440, 1520, 1535}^*$ with lesser number of degrees of freedom by factor $\beta_1 = 0.685$ in the presence of octet pseudogoldstones we have the ratio: $\frac{13.19\beta_1+3.76}{0.303\beta_1} = \frac{s_H^{eff}}{n_H^{eff}} \simeq 61.4$, i.e. the relations (24) are fulfilled. We have also the volume $V_H^{eff} = \frac{s_0 V_0}{s_H^{eff}} \simeq 274m_\pi^{-3}$, or the same $V_H^{eff} = \frac{N - \bar{N}}{n_H^{eff}} = \simeq 274m_\pi^{-3}$, where V_H^{eff} is effective volume of hadron part of mixed phase. We give now the result of calculations at RHIC energy. The calculation gives for nucleons only (without accounting of resonances): $\frac{s_H}{n_H} = \frac{10.335}{0.0195} \simeq 530$, for $n + \Delta$ this ratio is: $\simeq 282$, for the whole of nucleon resonance we have for this ratio: $\simeq 205$ and so on, i.e. $\frac{s_H}{n_H} \neq \frac{s_c}{n_c}$. But for the whole resonances the calculation gives: $s_H = 11.85$, $n_H = 0.0585$ in units m_π^3 . With lesser number of degrees of freedom by value $\beta_3 = 0.343$ (like for massive quarks) we have $\frac{s_H^{eff}}{n_H^{eff}} = \frac{11.85\beta_3+3.76}{0.0585\beta_3} \simeq 390$. Now the effective volume is: $V_H^{eff} \simeq \frac{16.3}{n_H^{eff}} \simeq 812m_\pi^{-3}$, or the same $\frac{s_0 V_0}{s_H^{eff}} \simeq 811m_\pi^{-3}$.

Hence the relations of type (24) are fulfilled also for ratio $\frac{s_H}{n_H}$ in hadron part of mixed but with effective number of degrees of freedom the same as for constituent quarks.

IV. THE BARYONS AND MESONS SPECTRA

We investigate the spectra for SPS and RHIC at thermal freeze-out. Apparently the favor results for heavy nuclei (for example Pb + Pb) correspond to thermal freeze-out temperature $T_f \approx 120$ MeV [31, 32]. We consider at first SPS energy. We find the volume V_f at $T_f = 120$ MeV from conservation of number net nucleons. The value μ_B^f at thermal freezing we find from

condition: $\frac{\bar{p}}{p}(T_c) = \frac{\bar{p}}{p}(T_f)$, where $\frac{\bar{p}}{p}(T_c) = e^{-2\mu_B/T_c}$. From here we have $\mu_B^f \simeq 174.35$ MeV. From formula (13) we have net nucleons density at $T_f = 120$ MeV, $\mu_B^f = 174.35$ MeV: $n_f^{N-\bar{N}} = 7.186 \times 10^{-3}m_\pi^3$. For $N_{net} \simeq 57$ we have $V_f = N_{net}/n_f \simeq 7950m_\pi^{-3}$. The value μ_s^f at 120 MeV may be found by no condition (25) (as the chemical equilibrium already is absent), but from ratio: $\frac{\bar{\Lambda}}{\Lambda}(T_c) = \frac{\bar{\Lambda}}{\Lambda}(T_f)$. From here we have found with accounting of various weak decays: $\mu_s^f \simeq 40 - 42$ MeV. From relations $\frac{n_{k+}}{n_{k-}}(T_c) = \frac{n_{k+}}{n_{k-}}(T_f)$ (with accounting of decays) we find for SPS the near value: $\mu_s^f = 41$ MeV [33]. One can now find by formulas of ideal gas the entropy density (at T_f) of nucleons and strange baryons: $s_B^f \simeq 0.125m_\pi^3$ and for nonstrange and strange mesons up to $m_i = 1820$ MeV: $s_{h+k}^f \simeq 1.3m_\pi^3$. Thus the entropy of baryons is $S_B^f = V_f s_B^f \simeq 992$, and the entropy of mesons $S_{h+k}^f = S_0 - S_B^f \simeq 2516$. The volume of mesons at thermal freeze-out is $V_{h+k}^f = \frac{S_{h+k}^f}{s_{h+k}^f} \simeq 1940m_\pi^{-3}$. We can estimate the number of π_0 : $N_{\pi_0} = V_{h+k}^f n_{\pi_0}$. For the value $n_{\pi_0} \simeq 0.0807$ (calculated without accounting of weak decays) we have $N_{\pi_0}^f \simeq 157$. The experiment per unit of rapidity gives $N_{\pi_0} = 165 \pm 20$ [34]. By analogy we find also the number of k_+ : $N_{k+} = n_{k+} V_{h+k}^f \simeq 0.0149 V_{h+k}^f \simeq 29$. Thus in this model we have different volume of freeze-out for baryons and mesons. The spectra of protons and strange baryons one should to calculate with V_f , and of mesons — with volume V_{h+k}^f .

The relations of type (24) must be fulfilled also for thermal freeze-out. Here we have $S_0 = S_f = s_{h+k}^f V_{h+k}^f + s_B^f V_f$, and for net nucleons: $n_1 V_0 = n_f V_f \simeq 57$, i.e. we have instead of (24) the relation: $\frac{s_0(T_0)}{n_1(T_0)} = \frac{s_{h+k}^f V_{h+k}^f}{n_f V_f} + \frac{s_B^f}{n_f} \simeq 44 + 17.4 = 61.4$. Thus the relations of type (24) are fulfilled (but now without of decrease of number of degrees of freedom). And backwards from conservation of entropy $S_0 = S_f$ one can to find the final volume V_f for baryons.

We calculate the spectra $p - \bar{p}$, Λ , Ξ , Ω direct and with resonance decays. We take into account the transverse flow. We use the hydrodynamic model with linear transverse velocity profile $v_\perp(r/R) \equiv v_\perp(x) = \frac{3}{2}\bar{v}_\perp x$, where \bar{v}_\perp is the averaged transverse flow velocity. The spectra of net protons expressed by formula:

$$\frac{dN^{p-\bar{p}}}{m_\perp dm_\perp} = V_f \frac{(e^{\mu_f/T_f} - e^{-\mu_f/T_f})}{\pi^2} m_\perp g_p \times \int_0^1 dx x I_0(p_\perp \frac{\sinh(\rho)}{T_f}) K_1(m_\perp \frac{\cosh(\rho)}{T_f}). \quad (26)$$

Here $\cosh(\rho) = \frac{1}{\sqrt{1-v_\perp^2(x)}}$, $\sinh(\rho) = \frac{v_\perp(x)}{\sqrt{1-v_\perp^2(x)}}$.

For Λ, Ξ, Ω we have factors $\exp(\frac{\mu_B - n\mu_s}{T_f})$, where

$n = 1, 2, 3$ correspondingly, and for k_+ - factor $V_{h+k}^f \exp(\frac{\mu_s}{T_f})$. We take into account also spectra at resonance decays, for example: $\Xi^0 \rightarrow \Lambda\pi^0$ (99.54%), $\Xi^- \rightarrow \Lambda\pi^-$ (99.88%), $\Xi_{1530} \rightarrow \Xi\pi$, $\Sigma_{1190} \rightarrow \Lambda\gamma$, $\Sigma_{1385} \rightarrow \Lambda\pi$, $\Omega^- \rightarrow \Xi^0\pi^-$.

At resonance decays we must take into account the resonance with transverse flow, and cascade decays also.

The resonance decays without transverse flow were investigated in the paper [35]. We have calculated here the some resonance decays spectra at SPS and RHIC energies. For example, spectra of Λ at decay $\Sigma^0 \rightarrow \Lambda\gamma$ is calculated thus:

$$\begin{aligned} \left(\frac{dN^\Lambda}{2\pi m_\perp dm_\perp} \right)_{y=0} &= \quad (27) \\ &= \frac{V_f}{4\pi^3} \frac{m_\Sigma b g}{p_1^* 4\pi} \int_0^1 dq \frac{T_f^2}{m_\perp} e^{\frac{\mu_B - \mu_s}{T_f}} f_{12}(p_\perp, q, \bar{v}_\perp) q. \end{aligned}$$

Here b - branching, g - stat.weight,

$$p_1^* = \frac{\sqrt{[(m_\Sigma + m_1)^2 - m_2^2][(m_\Sigma - m_1)^2 - m_2^2]}}{2m_\Sigma},$$

$m_1 = 1115$ MeV, $m_2 = 0$. We have also:

$$f_{12} = 2 \int_0^{z(p_\perp)} dy \frac{f_{11}(p_\perp, y, q, \bar{v}_\perp) \cosh(y)}{\sqrt{\cosh^2 y - (\frac{p_\perp}{m_\perp})^2}}. \quad (28)$$

Here $z(p_\perp) = \ln(\frac{\sqrt{E_1^2 + p_\perp^2 + p_1^*}}{m_\perp})$, $E_1 = \frac{m_\Sigma^2 + m_1^2 - m_2^2}{2m_\Sigma}$.

The value f_{11} in formula (28) is:

$$\begin{aligned} f_{11}(p_\perp, y, q, \bar{v}_\perp) &= \int_{x_1(p_\perp, y)}^{x_2(p_\perp, y)} dx x^2 e^{-x \cosh(y)} r(q, \bar{v}_\perp) \times \\ &\times \frac{I_0(\sqrt{(x^2 - (\frac{m_\Sigma}{T_f})^2)(r^2(q, \bar{v}_\perp) - 1)})}{\sqrt{(x_2(p_\perp, y) - x)(x - x_1(p_\perp, y))}}. \quad (29) \end{aligned}$$

Here

$$r(q, \bar{v}_\perp) = 1/\sqrt{1 - (\frac{3}{2}q\bar{v}_\perp)^2},$$

$$x_{1,2}(p_\perp, y) = \frac{m_\Sigma(E_1 m_\perp \cosh(y) \mp p_\perp \sqrt{E_1^2 + p_\perp^2 - m_\perp^2} \cosh^2(y))}{m_\perp^2 \cosh^2(y) - p_\perp^2}.$$

At cascade decays, for example $\bar{\Sigma}^0 \rightarrow \bar{\Lambda}\gamma$, $\bar{\Lambda} \rightarrow \bar{p}\pi$, we calculate at first the spectra of $\bar{\Lambda}$, however no at angle of 90 degrees (i.e. $y = 0$), but for the whole y , and then this spectra we use for calculation of spectra $\frac{dN^{\bar{p}}}{2\pi p_\perp dp_\perp}$. In spectra $\frac{d\bar{\Lambda}}{2\pi p_\perp dp_\perp}$ the value $\exp\{-x \cosh(y) r(q, \bar{v}_\perp)\}$ we replace now by the value $2K_1(x r(q, \bar{v}_\perp))$. Then the spectra $\frac{dN^{\bar{\Lambda}}}{m_\perp dm_\perp} \equiv f(m_\perp)$ we substitute to formula:

$$\begin{aligned} \left(\frac{dN^{\bar{p}}}{2\pi p_\perp dp_\perp} \right)_{y=0} &= \frac{m_\Lambda b g_\Lambda}{4\pi p^*} \frac{2m_\Lambda}{\pi m_{\bar{p}}^2} \times \quad (30) \\ &\times \int_0^{z(p_\perp)} \frac{dy}{\sqrt{\cosh^2(y_\Lambda) - \frac{p_\perp^2}{m_\perp^2} x_\Lambda(p_\perp, y_\Lambda)}} \int_{x_1(p_\perp, y_\Lambda)}^{x_2(p_\perp, y_\Lambda)} dy_1 \frac{y_1 f(m_\Lambda \sqrt{y_1^2 - 1})}{\sqrt{(x_2 - y_1)(y_1 - x_1)}}. \end{aligned}$$

We apply now to RHIC energy ($\sqrt{s}=130$ GeV). At measurement of antiprotons spectra in collaboration STAR were measured the spectra of direct \bar{p} and also spectra of \bar{p} from weak decays (with cascade). The our calculation shows that contribution of weak decays is $\simeq 30\%$. We give here the meanings of values, which were found for RHIC at $T_f = 120$ MeV (by analogy with SPS): $\mu_B^f = 35.3$ MeV, $\mu_s^f = 7.93$ MeV, $n_f^{N-\bar{N}} \simeq 1.062 \times 10^{-3} m_\pi^3$, $s_B^f \simeq 0.073 m_\pi^3$, $s_{h+k}^f \simeq 1.28 m_\pi^3$. From conservation of number net nucleons we have the volume V_f at $T_f = 120$ MeV without accounting of weak decays: $V_f = \frac{N-\bar{N}}{n_f} \simeq 15250 m_\pi^{-3}$. The entropy of baryons in this volume is $S_B^f = V_f s_B^f \simeq 1114$. The entropy of mesons is $S_0 - S_B^f \simeq 5236$. The mesons entropy density is $s_{h+k}^f \simeq 1.28 m_\pi^3$. From here $V_{h+k}^f \simeq 4090 m_\pi^{-3}$.

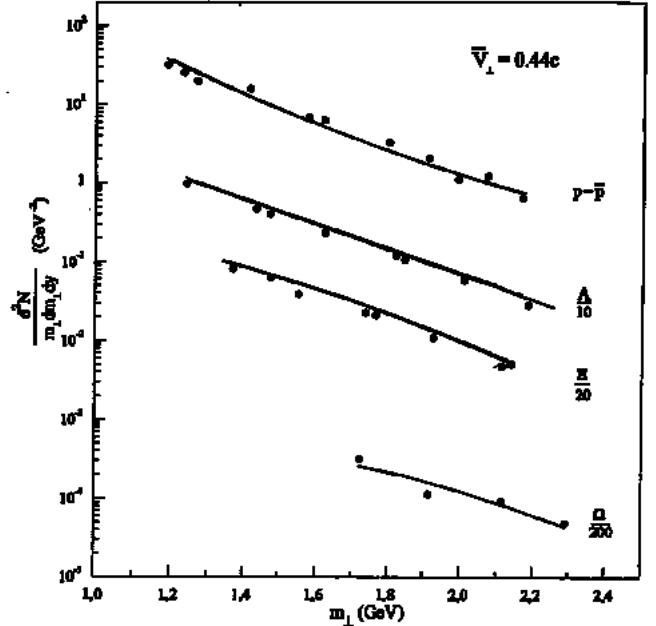


Figure 1: The baryons spectra in Pb+Pb collisions at SPS energy in central region of rapidity (quasiparticle model). The Λ - are direct and with resonance decays: $\Sigma^{1190} \rightarrow \Lambda\gamma$, $\Sigma^{1385} \rightarrow \Lambda\pi$, $\Xi^{1315} \rightarrow \Lambda\pi$. The Ξ^{1315} spectra - are direct + $\Xi^{1530} \rightarrow \Xi\pi + \Omega^- \rightarrow \Xi^0\pi^-$. The spectra $p - \bar{p}$ and Ω_{1672}^- - are thermal direct. The averaged transverse flow velocity $\bar{v}_\perp = 0.44c$. The experimental data are from [29, 39].

We have found the particles density at T_f : $n_{\pi^+}^f \simeq 0.067 m_\pi^3$, $n_{k^+}^f \simeq 0.0115 m_\pi^3$. This gives the estimation of number of particles: $N_{\pi^+} \simeq 274$, $N_{k^+} \simeq 47$. The value V_f^{h+k} we use for calculation, for example, of spectra k^- and π^+ for energy RHIC. One can also to show that relation of type (24) for thermal freeze-out are fulfilled and for RHIC energy.

One can calculate the contribution of weak decays into net nucleons density at $T_f = 120$ MeV (for RHIC) [33]: $n_w \simeq 2.96 \times 10^{-4} m_\pi^3$. With

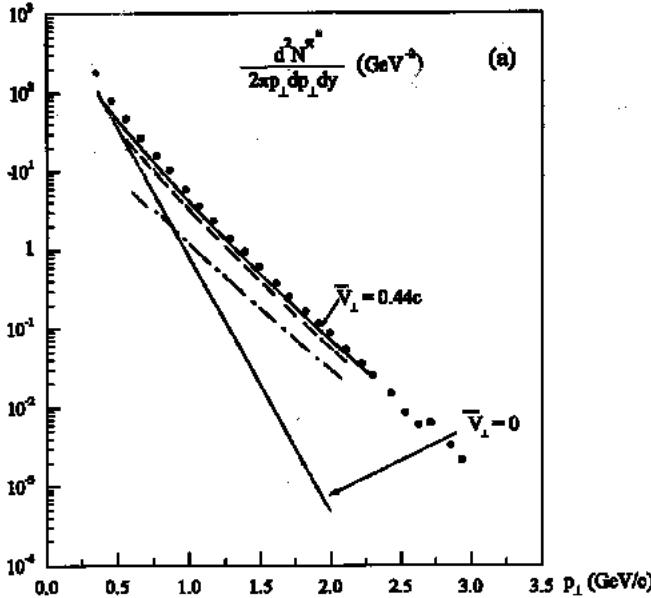


Figure 2a: The π^0 spectra (SPS, Pb+Pb) are direct + decays $\rho \rightarrow 2\pi + f_2 \rightarrow 2\pi$ ($\bar{v}_\perp = 0.44c$). The dashed line - the direct π^0 , the dash-dotted line — π^0 at decays. We show also the summary spectra at $\bar{v}_\perp = 0$. The data — from [34]).

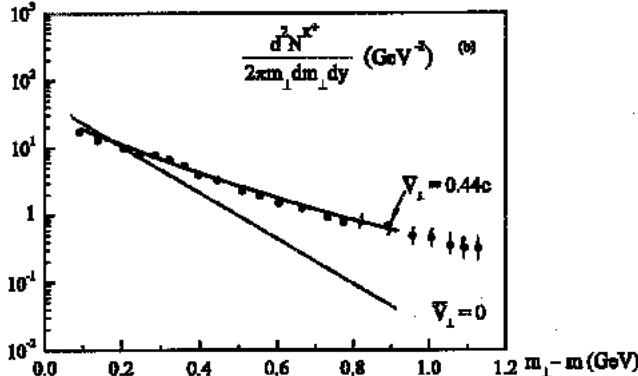


Figure 2b: The k^+ spectra (SPS, Pb+Pb) - direct + decay $K_{890}^* \rightarrow k\pi$ ($\bar{v}_\perp = 0.44c$). We show also the same spectra at $\bar{v}_\perp = 0$. The data — from [30]).

account of nucleons from decays we find equivalent volume V'_f : $n_{n-\bar{N}}V'_f = 16.3 + n_wV'_f$. From here $V'_f \simeq 21150m_\pi^{-3}$.

The spectra of direct and of daughter \bar{p} at weak decays we calculate by formulas (26)-(30) with factors type $V'_f \exp \frac{-\mu_B^f}{T_f}$, $V'_f \exp \frac{-\mu_B^f + \mu_s^f}{T_f}$, ... The contribution of weak decays with cascade decays to \bar{p} spectra gives $\simeq 30\%$.

The theoretical \bar{p} spectra and experimental spectra for central bin 0 – 6% (STAR Coll.) at mid rapidity are shown in Fig. 3a. In Fig. 3b, Fig. 3c we show also the direct π^+ spectra and with decay $\rho \rightarrow \pi\pi$ and the k^- spectra direct + $k_{892}^* \rightarrow k\pi$ and experimental data. In

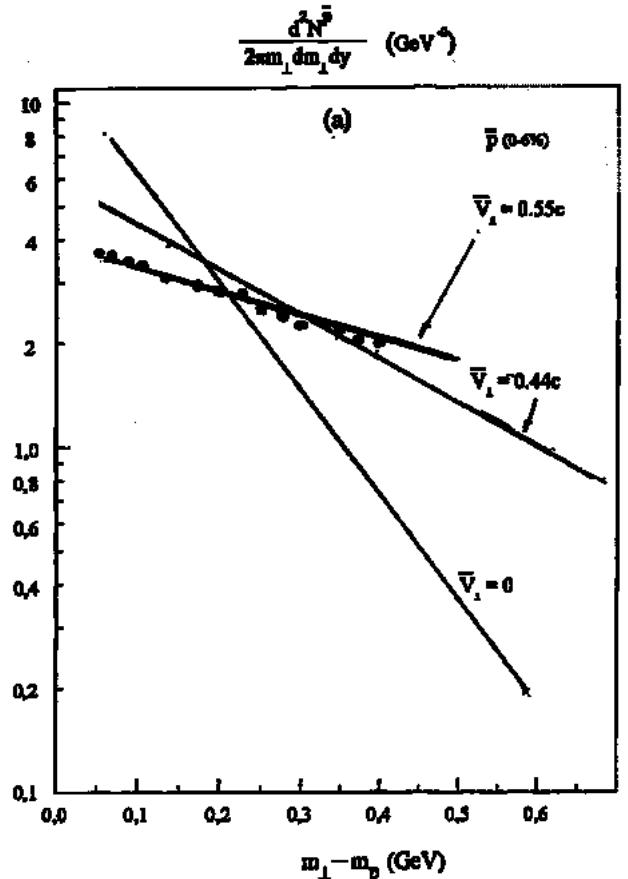


Figure 3a: The spectra \bar{p} (centrality 0 – 6%) at RHIC energy with accounting of weak decays ($\bar{v}_\perp = 0.55c$). We show also the same results with $\bar{v}_\perp = 0.44c$ and $\bar{v}_\perp = 0$ (quasiparticle model). The data from [22]).

Fig. 1 we show the baryons spectra: $p - \bar{p}$, Λ , Ξ , and Ω for SPS (at T_f) in central interval of rapidity. The π^0 and k^+ spectra are shown in Fig. 2a, Fig. 2b together with data. One can to see that spectra for RHIC have greater inverse slope than for SPS. With increasing mass of particle the inverse slope (i.e. the value \bar{v}_\perp) increases. Of course, one should note that obtained spectra with sufficiently large value of \bar{v}_\perp are correct only for not great values of p_\perp .

Thus, in the present quasiparticle model with isentropic evolution the normalization of baryons and mesons spectra do not defined by free parameteres, but defined by conditions in initial state (T_0 , V_0), i.e. by quark-gluon plasma.

V. THE CALCULATION BY THE USE OF PERTURBATIVE THEORY

We had investigated the physical characteristic of initial and mixed phases and some baryons and mesons spectra for SPS and RHIC in effective quasiparticle model with isentropic evolution. In this section we

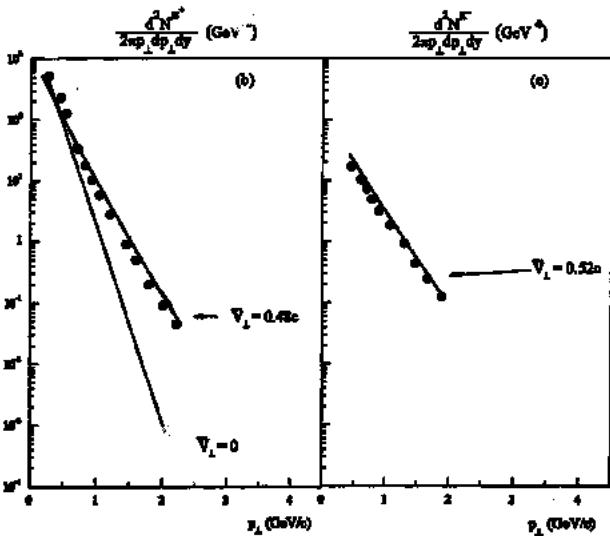


Figure 3b: The π^+ spectra (RHIC) - direct + decays $\rho \rightarrow \pi\pi$ ($\bar{v}_\perp = 0.48c$). It is shown the same spectra with $\bar{v}_\perp = 0$.

Figure 3c: The k^- spectra (RHIC) - direct + decays $K_{892}^* \rightarrow \kappa\pi$ for $\bar{v}_\perp = 0.52c$. The data — from [22, 40].

shall consider at first the analogous problem of nuclear collisions by the use of ordinary perturbative decomposition of thermodynamic values in powers of running coupling $\alpha_s(T)$ in the form [41]: $\alpha_s(T) = 6\pi/(11N_c - 2N_f) \ln((2m_s + 3T)/\lambda)$ up to order $O(\alpha_s)$ with QCD parameter $\lambda \sim 0.2$ GeV. Here the value Q^2 in coupling $\alpha_s(Q^2)$ is considering as the average of Q^2 in S-canal at temperature T . For example, the entropy density for gluons in plasma is: $s_g = 32/45\pi^2 T^3 (1 - \frac{15\alpha_s}{4\pi})$ and so on. We calculate here also the values T_0, V_0, S_0 .

The initial temperature T_0 can be find from formula analogous (23), where we have now:

$$D_0(m_q, \mu_q, T_0, m_s, \lambda) = 1. \quad (31)$$

Here the values $d_2(m_q, \mu_q, T_0, \lambda)$ and $n_{02}(m_q, \mu_q, T_0, m_s, \lambda)$ are extended up to order $O(\alpha_s)$. We find from here for $\alpha = 1.1, \mu_B = 247$ MeV, $m_q = 0, \lambda = 180$ MeV the such meanings(for SPS): $T_0 = 178$ MeV, $\alpha_s(T_0) = 0.454$, $s_0(T_0) \simeq 26.4m_\pi^3$, $s_c(T_c) \simeq 22.4$, $n_1(T_0) = 0.475$, $n_1(T_c) = 0.426$ in units m_π^3 , $V_0 = N_0/n_1(T_0) \simeq 120m_\pi^{-3}$, $S_0 = s_0V_0 = 3180$ (where $N_0 \equiv (N - \bar{N}) \simeq 57$).

In this model the entropy of nucleons and strange baryons is the same as in quasiparticle model: $S_B^f = 992$, the entropy of mesons $S_{h+k}^f = S_0 - S_B^f \simeq 2200$ and $V_{h+k}^f \simeq 1700m_\pi^{-3}$. We have here $N_{\pi^0}^f \simeq 136$, $N_{k^+} \simeq 25$.

For RHIC energy we find for $\alpha = 1.11, \mu_B = 50$ MeV, $m_q = 0, \lambda = 180$ MeV the meanings: $T_0 \simeq 219$ MeV, $\alpha_s(T_0) = 0.418, s_0(T_0) = 50.11m_\pi^3, n_1(T_0) = 0.145$,

$n_1(T_c) = 0.084m_\pi^3, V_0 = N_0/n_1 \simeq 111.7, V_c \simeq 193m_\pi^{-3}, S_0 = s_0V_0 \simeq 5600$. We have here $\tau_0 \simeq 2.16$ fm, $\tau_c \simeq 3.73$ fm. One can show, that in constituent quarks phase the relations (24) here also are fulfilled only with decrease of number of degrees of freedom in the presence of pseudogoldstone states.

We have here by analogy with quasiparticle model $V_f \simeq 15250m_\pi^3$, the entropy of baryons $S_B^f \simeq 1114$, the entropy of mesons is $S_{h+k}^f = S_0 - S_B^f = 4486$ and $V_{h+k}^f = \frac{S_{h+k}^f}{s_{h+k}^f} = 3500m_\pi^{-3}$ ($s_{h+k}^f \simeq 1.28m_\pi^3$). That gives the estimation: $N_{\pi^+} \simeq 243, N_{k^+} \simeq 40$.

In this simple model we have not great decrease of normalization of mesons spectra at SPS and RHIC (by 12-13 %) in comparison with quasiparticle model, i.e. apparently it takes place within the limits of precision of measurement. Thus we do not have here noticeable difference for spectra of particles in comparison with quasiparticle model. However this model disagrees with $SU(3)$ lattice data in region of phase transition. The values $s/T^3, \epsilon/T^4, p/T^4$ decrease very weakly in this case (For example, at $3T_c \rightarrow T_c$ the value s/T^3 decrease only by value $\simeq 0.78$) The coupling strength $\alpha_s(T)$ increase (although not great) at $T \rightarrow T_c$ from above (unlike effective quasiparticle model). Thus we see, that spectra of particles weakly depend on character of phase transition $QGP \rightarrow$ hadrons.

But one can show [12], that in this model we have too great energy loss ΔE_g of gluon jet at RHIC energy: $\sim 80 - 90\%$. That is caused in the main by value of effective coupling strength.

Let us consider now the quasiparticle model with phenomenological parametrization of running coupling $G(T)$ (8) We use here the formulas of type (10)-(13) and of type (14)-(18) for calculation of thermodynamic values m_q, s_0 and n_0 , but with coupling $G(T)$ (8) and with $C(T, T_c) = 1$. With help of corresponding parameters T_s, λ it is possible well to fit the new lattice data even near of T_c . For calculation of initial conditions we use the equation of type (23). For example, for possible parametrization [6]: $\lambda = 5.3, T_s/T_c = 0.73$ (at $N_f = 3$) we find for RHIC energy ($\sqrt{s} = 130$ GeV) the following meanings: $T_0 \simeq 250$ MeV, $G(T_0) \simeq 2.533$ (i.e. $\alpha_s(T_0) \simeq 0.509$) and also $\alpha_s(T_c) \simeq 1.948$. We find corresponding values for entropy and net nucleons density: $s_0(T_0) \simeq 84.36m_\pi^3, s_c(T_c) \simeq 12.96m_\pi^3, n_1(T_0) \simeq 0.210m_\pi^3, n_1(T_c) = 0.0442m_\pi^3$. From here we have the estimation for initial state: $\tau_0 \simeq (N - \bar{N})/(n_1(T_0)\pi R_{Au}^2) \simeq 1.5$ fm., and by analogy $\tau_c \simeq 7.14$ fm. We have for complete entropy: $S_0V_0 = 6548$, i.e. only on $\sim 3\%$ the difference from its value in effective quasiparticle model. Thus we have practically identical baryons and mesons spectra in these two models.

However we have calculated the jet quenching in plasma in this model by analogy with effective quasiparticle model [12]. The result is dramatic: the energy loss ΔE_g of gluon jet (for two parametrization of λ and T_s/T_c) four - five times as much than energy E_g of itself jet!. This contradiction is caused by too great value of

running coupling $G(T)(8)$ in this model, which besides increase at $T \rightarrow T_c$ from above.

VI. CONCLUSION

In this paper we investigate the isentropic evolution of expanding quark-gluon plasma with phase transition at first into plasma part and then hadron part of mixed phase at temperature T_c , and further isentropic expansion up thermal freeze-out at temperature T_f . For investigation of initial plasma phase we use the quasiparticle model, which allow apparently to sum up partially the perturbative expansion of thermodynamic function of interacting plasma in powers of coupling constant. In the papers [4, 6] was used the phenomenological parameterization of coupling constant in quasiparticles model in accordance with lattice data. However close to T_c the coupling constant is sufficiently great, therefore in the vicinity of the phase transition the perturbative methods are not expected to be reliability.

In the paper [10] was shown, that at such parameterization the thermal mass of gluon grows in the vicinity of the phase transition when $T \rightarrow T_c$ from above. This do not agrees with SU(3) lattice data for Debye mass. In the paper [10] was considered the phenomenological model of confinement, where the decrease of thermodynamic values at $T \rightarrow T_c$ from above is caused no increase of mass, but by decrease of the number of active degrees of freedom (i.e. by modification $g_g \rightarrow C(T)g_g$). It is supposed, that behaviour of thermal gluon mass $m_g(T)$ is similar with behaviour of Debye mass, i.e. it decreases with $T \rightarrow T_c$ from above. This gives the good description of SU(3) lattice data. The similar modification is used also for quasiparticle model with quarks.

It is assumed that Debye gluon mass have form $m_D(T) = \tilde{G}(T)T$ and it has a small gap at $T = T_C$ in accordance with SU(3) lattice data. It is assumed also that the proportionality of type (5) between m_D and thermal gluon mass m_g remains true in the vicinity of phase transition.

We use such model for description of isentropic evolution in common case of quasiparticles with $\mu_B \neq 0$ for calculation of physical characteristic of initial plasma phase: T_0 and V_0 . In result we have obtained the formula (23) for definition of these values, where we use the approximation (20) for $\left(\frac{dN}{dy}\right)_{A+A}$ and experimental meaning of average number of net nucleons $(N - \bar{N})$ in central region of rapidity at SPS and RHIC energies. From conservation of entropy and of number of net nucleons follow also the relations (24), which must be fulfilled simultaneously with (23). The relations (23), (24) depend on effective coupling constant G_0 , which is determined by asymptotic value of thermal gluon mass at $T = 3T_c$ [13]. For that we use the asymptotic value of running coupling $\alpha_s(T)$ [6]. In result were obtained the initial meaning T_0 , V_0 at SPS and RHIC energy: for SPS we have $T_0 \simeq 175$

MeV, $V_0 \simeq 518 \text{ fm}^3$, (i.e. $\tau_0 \simeq 3.28 \text{ fm}$) and $V_c \simeq 650 \text{ fm}^3$ (i.e. $\tau_c \simeq 4.1 \text{ fm}$), and for RHIC $T_0 \simeq 219.6 \text{ MeV}$, $V_0 \simeq 322 \text{ fm}^3$ and $V_c \simeq 965 \text{ fm}^3$, i.e. $\tau_0 \simeq 2.2 \text{ fm}$, $\tau_c \simeq 6.5 \text{ fm}$. We have also $S_0 \simeq 3508$ and $\simeq 6350$ for SPS and RHIC correspondingly.

When the temperature is lowered ($T \rightarrow T_c$), more and more gluons form heavy glueballs, and the weaker becomes the interaction. This state can be interpreted as superposition of nonperturbative multigluon and weak glueball exchange [10]. The interaction between the quarks also becomes weaker. It is possible to interpret, that there is appearance of massive constituent quarks, while the multigluon state is absorbed into constituent quarks masses.

Further we show, that massive constituent quarks appears with decrease of number of degrees of freedom in the presence of octet of pseudogoldstone states. That follow from conservation of entropy and of number of net nucleons. The hadron part of mixed phase appears also with decrease of number of degrees of freedom — the same as for constituent quarks. However at thermal freeze-out (at $T = T_f$) hadrons appears already with normal number of degrees of freedom.

Thus we have at SPS the short plasma phase. With above-mentioned parameters we have calculated in effective quasiparticle model the some typical baryons and mesons spectra with account of resonance decays. We use for heavy nuclei the temperature of thermal freeze-out $T_f = 120 \text{ MeV}$ (as in "standard" model [31, 32] The normalization of baryons and mesons spectra in the present model is no free parameter, but defined by initial condition in quark-gluon plasma.

One should note that there are the models for description of particle production with simultaneous chemical and thermal freeze-out [36], where 2-3 free parameters describe well the spectra at RHIC energy. In the present work we use "standard" scenario with two freeze-out - chemical and thermal.

The main object of present work is no study the whole spectra at SPS and RHIC, but the study of dependence of spectra from initial state and from phase transition stage. For this aim we consider in Sec.V the analogous isentropic problem by the use of ordinary perturbative theory up to order $O(\alpha_s(T))$, where $\alpha_s(T)$ is running coupling. We do not find in this approximation of noticeable difference for particles spectra in comparison with effective quasiparticle model, although this model disagrees with SU(3) lattice data in region of phase transition (unlike quasiparticle model). However it can be shown [12] that in this model we have the too great energy loss of gluon and quark jets in plasma ($\sim 80 - 90\%$), that contradict to suppression of π^0 -spectra, reported by PHENIX [40].

In this section we consider also the quasiparticle model with phenomenological parametrization of coupling $G(T)$, which gives good fit of new lattice data. We investigate here also the initial condition in plasma at RHIC energy (on the basic of equation of type(23)). We

find that baryons and mesons spectra in this model practically do not differ from spectra in effective quasiparticle model.

However the situation with energy loss in plasma here yet more dramatic - the energy loss of gluon jet exceed the energy of itself jet. Thus apparently we have correct quantitative description of energy loss of gluon and quark jets in plasma only in considering here effective quasiparticle model.

But of course the question about nonperturbative solution for thermodynamic values in quark-gluon plasma remains open (for example, for thermal mass of quasiparticles and running coupling and so on). Unfortunately in the vicinity of the phase transition is used the phenomenological parameterization for these values.

In this model we do not consider also the intrinsic

volume of particles. We had investigated in the main the ratios of thermodynamic values. We use the models [37, 38], where effects due to excluded volume correction cancel out in ratios.

At calculation in central region of rapidity we do not take into account the difference of number protons and neutrons.

It should be noted also the decrease of protons suppression for moderate p_\perp at RHIC energy [40] It is possible to use for this problem the hydrodynamic spectra in quasiparticle model for p and \bar{p} with accounting of resonance decays.

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